Watching Buildings Go Up

Photograph 1
Bay Adelaide Centre, Toronto

"Mathematical Lens" uses photographs as a springboard for mathematical inquiry. The goal of this department is to encourage readers to see patterns and relationships that they can think about and extend in a mathematically playful way.

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The topping-off ceremony (a celebration held when a building under construction reaches its highest point) was held for the Bay Adelaide Centre in downtown Toronto (**photographs 1** and **2**) on Tuesday September 23, 2008. The building is 715 feet tall. The work at this site involved a number of steps, starting with removal of an existing building and followed by construction of the underground portion of the new building.

1. (*a*) Use the information in **photograph 3** to calculate when the construction reached ground level. Assume that no work was done on weekends.

   (*b*) **Photograph 2** was taken late in the day on Thursday September 11, 2008. If we assume a constant rate of progress, how high was the building on that date?

2. As in most major cities in North America, the traffic in Toronto is terrible; drivers often find themselves inching along at an average speed of 5 mph. The sign in **photograph 3** claims that the progress of the construction is faster than the movement of Toronto traffic. If we assume that the speed of Toronto traffic is 5 mph, is this claim true?

3. (*a*) Suppose that on the day construction of the Bay Adelaide Centre reached ground level, a businesswoman in the nearby 907-foot-high Scotia Plaza building (in **photograph 2,** the building with the red *S*) looked down at the construction site from her office, located 630 feet above ground level. At ground level, Scotia Plaza is 100 feet away from the Bay Adelaide Centre. Find the angle (y in **fig. 1**) that the businesswoman would have to tilt her head down from the horizontal (i.e., the angle of depression) to see the construction site.

   (*b*) Suppose that this person decided to look at the top of the Bay Adelaide Centre once a week. Find the value of y (see **fig. 2**) for each week from the first day she looked until the day of the topping-off ceremony. Describe how y changes during this period. Would there have been times when she would have noticed a big change in y within one week?

   (*c*) Sketch the graph of y versus time. By using the graph and one other method. find the time when y is equal to 45°.

4. (*a*) The official Web site for the Bay Adelaide Centre (www.bayadelaidecentre.com) states that the building has 8 corner offices per floor. Consult **photographs 1** and **2** and show how the building's design has made possible this many corner offices per floor.

   (*b*) Design a layout for a building that has more than 8 corner offices per floor. What would the footprint of this building look like?

5. (*a*) One safety feature of the Bay Adelaide Centre is that the width of the stairwells is 53 inches, 20 percent wider than what Ontario's building code requires. What is the minimum width that Ontario requires?

   (*b*) What is the width of a stairwell in your school? Do all the stairwells have the same width? How does the width of the stairwells in your school compare with that in the Bay Adelaide Centre?

Photograph 2
Toronto skyline with Bay Adelaide Centre under construction

Photograph 3
Sign on the Bay Adelaide Centre

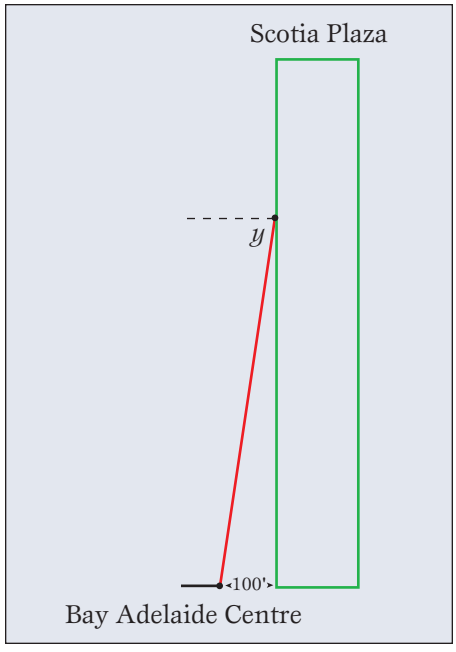

Fig. 1 The angle of depression from an office in the Scotia Plaza building to the construction site

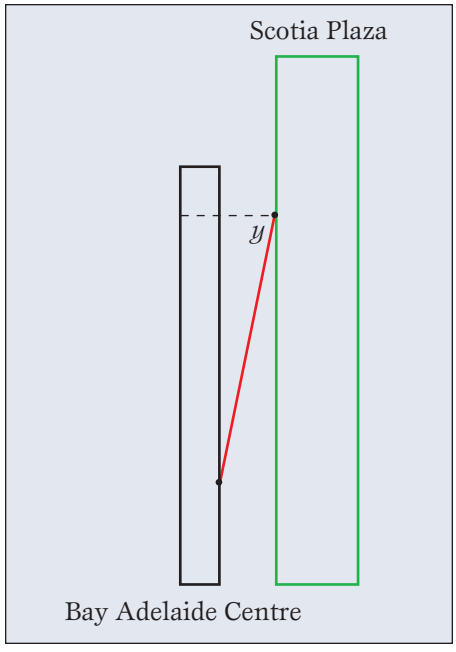

Fig. 2 As the new building rises, the angle of depression changes.

MATHEMATICAL LENS solutions

1. (*a*) At a construction rate of 4.5 feet per day, it would take almost 159 days—715/4.5—to reach the day on which the topping-off ceremony took place (September 23, 2008). By consulting a calendar for 2008 and working backward, we can see that construction of the Bay Adelaide Centre reached ground level on February 14, 2008.

(*b*) There are 8 working days (weekdays) between September 11 and September 23, 2008. Since $8 \times 4.5 = 36$, the building would have been 679 feet tall on September 11.

2. 5 miles/hour = 120 miles/day
= 633,600 feet/day

At this rate, the Bay Adelaide Centre would be finished in less than a day! So it is not true that the rate at which the building is going up is faster than the speed of Toronto traffic.

3. (*a*) Since

$$\tan^{-1}\left(\frac{630}{100}\right) \approx 81°,$$

the businesswoman would have to tilt her head at an angle of almost 81°.

(*b*) Let x be the number of weeks that the businesswoman has been looking at the top of the Bay Adelaide Centre. Since the building rose 22.5 feet in 5 days, then

$$y = \tan^{-1}\left(\frac{|630 - 22.5x|}{100}\right).$$

Use of the absolute value function is needed, because after 28 weeks the businesswoman would have looked straight out at the top of the building and on subsequent days she would have had to look up.

The values of y for each week up to the day that the building reached its highest level can be determined by using a TI-84 calculator (see **figs. 3a** and **3b**) or MS Excel (see **table 1**). As the data show, the change in y was quite small at first and then very large as the businesswoman's office. At that time, she would have noticed a dramatic change in the angle.

(*c*) See **figure 4** for the graph of y versus time.

The time when y is equal to 45° can be found by locating the point of intersection between the graphs of

$$y = \tan^{-1}\left(\frac{|630 - 22.5x|}{100}\right)$$

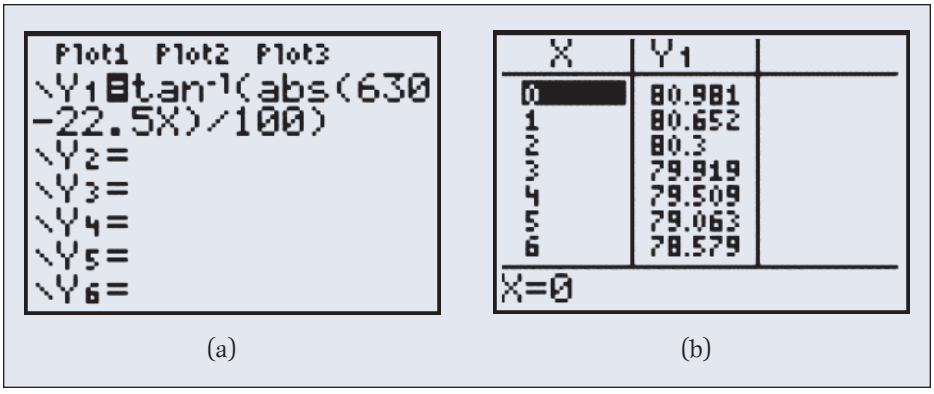

Fig. 3 The y = menu (**a**) and the table output (**b**) for angles of depression and elevation to the top of the construction

Table 1

Weekly Angles of Depression or Elevation

x (weeks)	y (degrees)
0	80.98
1	80.65
2	80.30
3	79.92
4	79.51
5	79.06
6	78.58
7	78.05
8	77.47
9	76.83
10	76.13
11	75.35
12	74.48
13	73.50
14	72.39
15	71.13
16	69.68
17	68.00
18	66.04
19	63.72
20	60.95
21	57.59
22	53.47
23	48.37
24	41.99
25	34.02
26	24.23
27	12.68
28	0.00
29	12.68
30	24.23
31	34.02

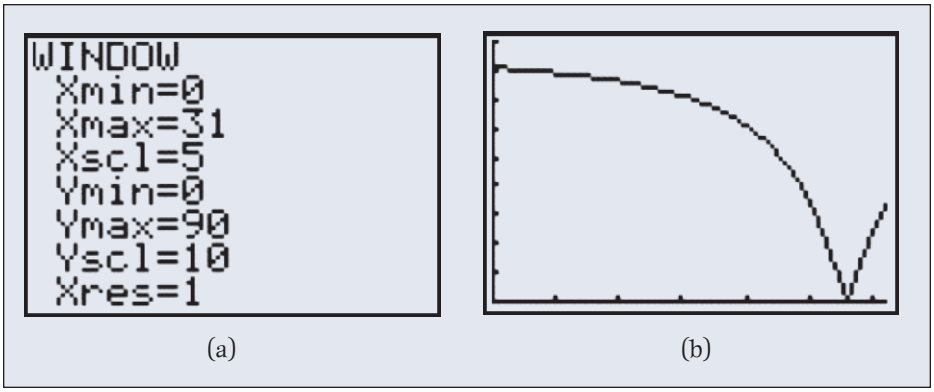

Fig. 4 Window settings (**a**) and graph (**b**) for the function in **figure 3a**

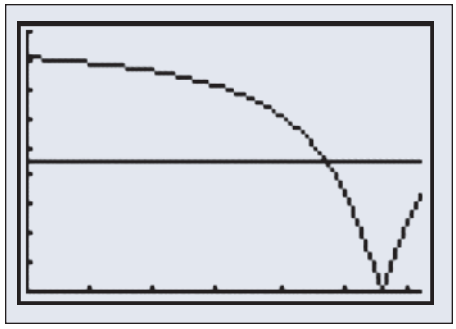

Fig. 5 Intersection of the inverse tangent function with the line $y = 45$

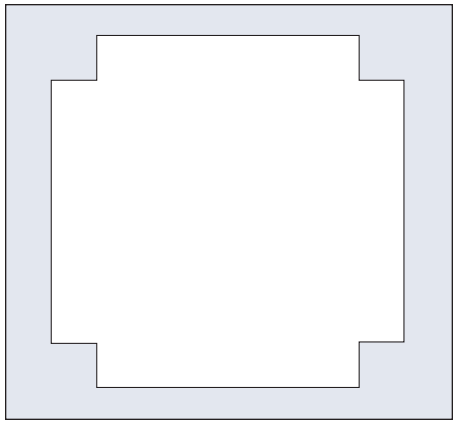

Fig. 6 The footprint of the Bay Adelaide Centre

and

$$y = 45°$$

(see **fig. 5**). Alternatively, this time can also be found by solving the following equation:

$$\tan^{-1}\left(\frac{|630 - 22.5x|}{100}\right) = 45°$$

$$\frac{|630 - 22.5x|}{100} = \tan(45°)$$

$$\frac{|630 - 22.5x|}{100} = 1$$

$$x \approx 23.6 \text{ or } x \approx 32.4$$

Both approaches show that the angle will be 45° after about 24 weeks. The larger value of x was discarded since the topping-off ceremony took place after about 31 weeks.

4. (*a*) Many office workers love having a corner office because of the view. Extra corner offices for the Bay Adelaide Centre can be created by modifying the corners (see **fig. 6** for the footprint of the Bay Adelaide Centre).

(*b*) Diagrams will vary. **Photograph 4** shows a building that has 3 corner offices per corner. **Photograph 5** shows a Toronto building that also has many corner offices per corner.

5. (*a*) According to Ontario's building code, the minimum width of a stairwell is about 44 inches.

(*b*) Answers will vary. ∞

Members who wish to use this month's photographs in a classroom setting can download the image from NCTM's Web site, www.nctm.org. Follow links to *Mathematics Teacher,* and choose Current Issue. Then select Mathematical Lens from the Departments, and look for the link to the image.

Photograph 4
Rockefeller Research Laboratories, New York City

PHOTOGRAPH BY BRIGITTE BENTELE; ALL RIGHTS RESERVED

Photograph 5
A Toronto office building

PHOTOGRAPH BY RON LANCASTER; ALL RIGHTS RESERVED