

MY FAVORITE
lesson

Anna Spanik

Building the Unit Circle: A Patty Paper Approach

My favorite lesson evolved from a session at the 1995 NCTM Annual Meeting in Boston. We were a group of eight junior and senior high school teachers from Nova Scotia, most of us attending our first NCTM Annual Meeting. We planned our days to maximize exposure to sessions and would share our experiences every evening.

A member of our group attended a workshop on Patty Paper® Geometry by Michael Serra and purchased the book by Key Curriculum Press. We took these investigations to our grade ten classrooms with great success and even initiated some professional development within our district.

We were delighted with how thoroughly engaged students were in the Patty Paper tasks and with the rich opportunities for clarifying language and notation. Although I still use this investigative approach in teaching geometry, my favorite lesson involves using Patty Paper to teach the unit circle. Although we teachers see the wonderful symmetries in the unit circle, students do not necessarily find these connections obvious.

THE DAY BEFORE

Students are given an activity sheet that asks them to determine the exact values of the sine, cosine, and tangent of angles whose measures are 30° , 45° , and 60° . We then engage in full-class discussion to ascertain that the coordinates of any point on the unit circle are $(\cos(a), \sin(a))$ where a is the angle in standard position. Students will use these notes throughout the investigation. To become familiar with Patty Paper, they draw an angle ABC , either acute or obtuse, and bisect it while we engage in discussion about folding carefully and accurately.

THE INVESTIGATION

Students work in groups of two or three. The investigation takes approximately sixty minutes.

Getting Started

Students are given a sheet of Patty Paper on which a large circle is drawn; they fold an x - and a y -axis so that the origin is in the center of the circle. We identify this as a unit circle, and students determine the x - and y -intercepts. We interpret these coordinates in terms of 0° , 90° , 180° , 270° , and 360° angles. This process gives us a chance to talk about the language we are using and to reinforce the implications of $r = 1$.

Angles Related to 45°

Students have no difficulty constructing a 45° angle in the first quadrant by bisecting 90° . Our discussion reinforces important terminology, and we use the activity sheet from the previous day to record the coordinates and consider their meaning. Students use folding and their 45° angle to create a 135° angle and label the coordinates. Meanwhile, I record results on the board and engage students in discussion about their method of folding, how they know the measure of the angle, and how they can be sure of the coordinates. Students complete the coordinates for the points where the terminal sides of angles of 225° and 315° intersect the circle.

Angles Related to 60°

Using a fresh Patty Paper with a circle drawn, students construct a 60° angle in the first quadrant, label the coordinates, and use folding to complete angles of 120° , 240° , and 300° . Folding a 60° angle presents a challenge, and students

may need some hints; this challenge gives us a chance to review the symmetries that exist in an equilateral triangle.

Angles Related to 30°

Bisecting the 60° angle gives a 30° angle, from which students can determine the coordinates for the points where the terminal sides of angles of 150° , 210° , and 330° intersect the circle.

Finishing Up

Students transfer their coordinates to a master copy for homework. Nevertheless, they tend to keep the Patty Paper versions and are quite proud of them.

A FINAL WORD

I love this lesson because my students enjoy it, they are thoroughly engaged regardless of their previous experience, and they recognize patterns for themselves. Further, this lesson plants the seeds for subsequent work with related angles and solving trigonometric equations.

"The Back Page" provides a forum for readers to share a favorite lesson. Lessons to be considered for publication should be submitted to mt.msubmit.net. Lessons should not exceed 600 words and are subject to abridgment.



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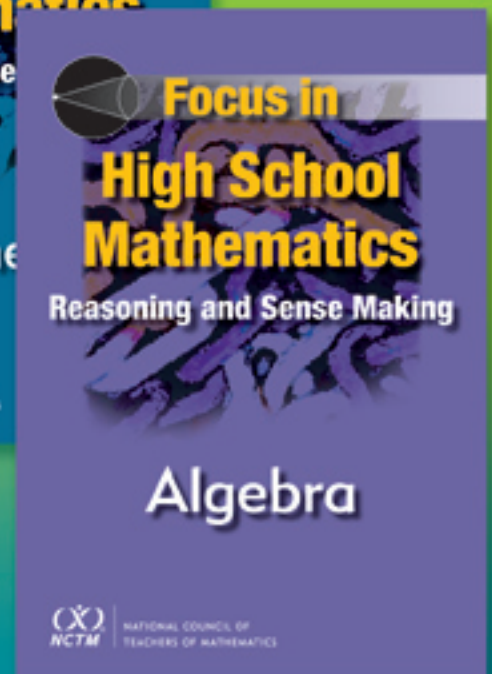
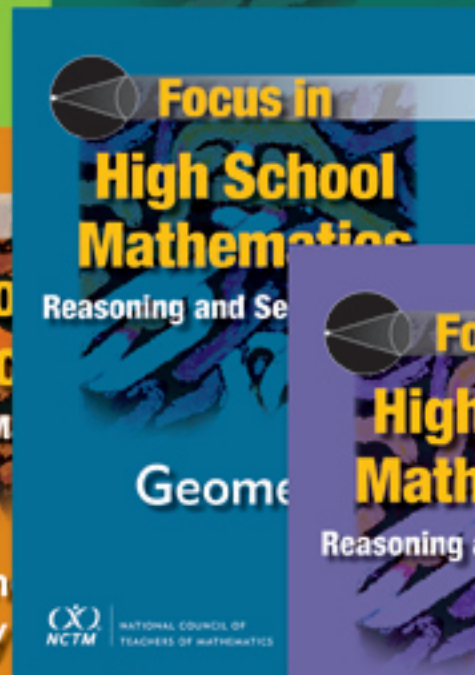
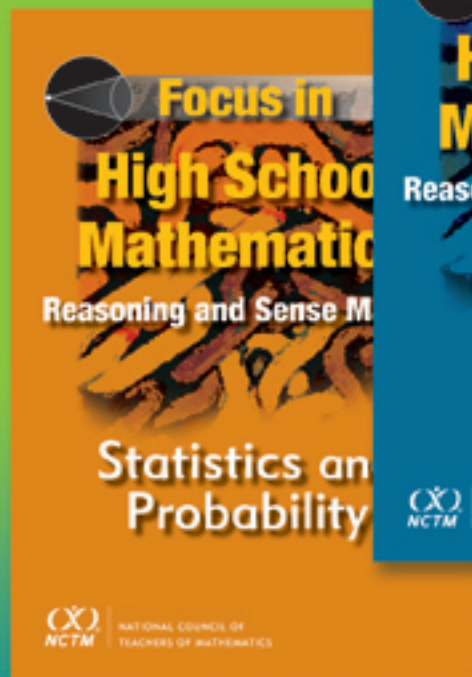
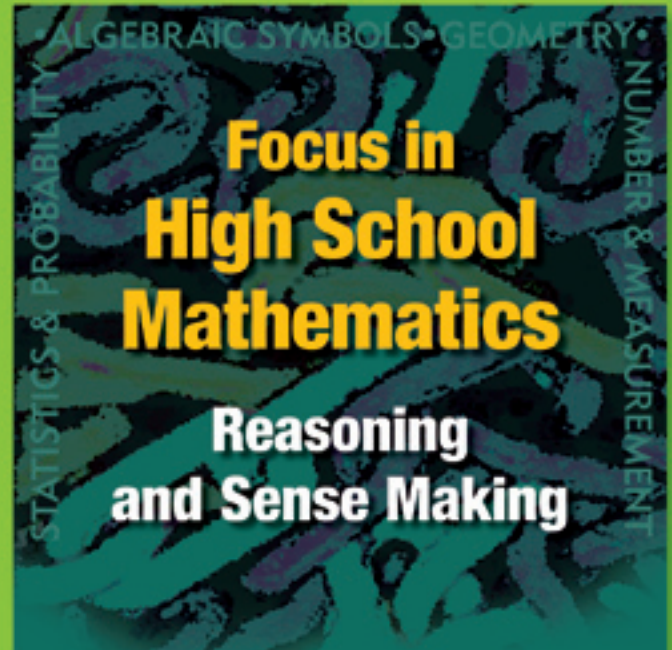
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Column A: Percentage of students with the correct answer before Hands-On Equations®

Column B: Percentage of students with the correct answer after 7 lessons of Hands-On Equations

Grade 8, n=76. Study #159MA: 8th grade students in an inner city middle school including 21 LD students and 22 ELL students

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